

$$\frac{dp}{dr} = \frac{c}{2(r-a)^{\frac{3}{2}}} = \frac{\Delta p}{2(r-a)}$$

and substituting for $\frac{dp}{dt}$ we get

$$\frac{r-a}{\frac{dr}{dt}} = \frac{\Delta p}{2 \frac{dp}{dt}}$$

But, neglecting a , the left hand term in this equation is the time in hours that the central calm will take to reach the station; so that,

$$\text{Time of arrival} = \frac{\Delta p}{2 \frac{dp}{dt}} \quad (3)$$

In words, *the time of arrival is the fall below mean divided by twice the rate of fall.*

If, therefore, we find that our series of observations agree in indicating the same time of arrival, there can be no doubt but that the cyclone is directly approaching.

As an example of equation (3), we have already seen that by Table 1, $\frac{dp}{dt}$ was 0.083; and as Δp was 0.32 at that hour, the computed time of arrival is 9 a. m., which is quite correct.

In order to add to this series of cases, let us take the cyclone which passed over Kingston, Jamaica, August 18, 1880. This was before the weather service was established there, so that Kingston was really an isolated station. The cyclone approached from the Windward Islands, and, according to the chart in Meteorological Observations, Vol. I, the cyclone center was not directly approaching till noon, when it turned on its course and made straight for Kingston.

TABLE 5.—Kingston, August 18, 1880.

Hour.	p .	Δp .	r .	Δp computed.	$\frac{dp}{dt}$	Time of arrival, as computed
	<i>Inches.</i>	<i>Inches.</i>	<i>Miles.</i>	<i>Inches.</i>	<i>Inches.</i>	
7 a. m.	29.782	0.14	256			
8 a. m.	29.744	0.18	220		0.017	2:00 p. m.
11 a. m.	29.714	0.21	184		0.014	6:30 p. m.
1 p. m.	29.687	0.24	148	0.22	0.012	11:00 p. m.
3 p. m.	29.666	0.26	112	0.26	0.022	9:00 p. m.
5 p. m.	29.597	0.32	76	0.32	0.060	8:00 p. m.
7 p. m.	29.428	0.49	40	0.49	0.181	8:30 p. m.
9 p. m.	28.874	1.05	4½	*1.05		

* Measured along the tangent to the curve.

The velocity of the center was 18 miles an hour, and p_m was taken to be 29.922.

The computed times of arrival before 3 p. m. are irregular; this shows that the center was *not* directly approaching. The times subsequently agreed, which shows that then the center *was* directly approaching. The time of arrival computed by equation (3) was 8:30 p. m. and the calm center actually arrived at 9 p. m.

From noon onward, then, equation (1) should hold good, and we easily find

$$a = +12$$

$$c = 2.6$$

whence result the computed values of Δp given in Table 5, and in fig. 4, Plate I, in which the limiting values of φ and the gradient are 57° and 0.015.*

We will now consider the case of two stations in the Tropics, on the line of usual approach of hurricanes, and connected by telegraph.

Let Δp be the fall below mean at the station nearer the hurricane at the time t ; and let t' be the time at the further station when the fall below mean reaches the same value Δp ; then

* This small gradient is surprising, as to my own knowledge the wind reached full hurricane force.

if D be the distance between the two stations in miles, the hourly velocity of the center toward the stations will be

$$\frac{dr}{dt} = \frac{D}{t' - t} \quad (4)$$

when the interval $(t' - t)$ must be expressed in hours and decimals of an hour.

Equation (4) is, of course, independent of (1), and holds good for any relation between Δp and r , provided that Δp increases as r decreases.

Hence, by the mutual exchange of barometer readings by telegraph, each station may come to know $\frac{dr}{dt}$, and hence predict the time of arrival of the center.

Take for example the cyclone of 1886 (Tables 2 and 3). At 7 a. m. on the 19th Δp at Kingston was 0.14; but Δp did not reach 0.14 at Montego Bay until 4 p. m. And as $D = 83$ miles, we have $\frac{dr}{dt} = \text{velocity of center} = 9$ miles an hour.

Similarly for $\Delta p = 0.17$, we get $\frac{dr}{dt} = \text{velocity of center} = 17$ miles an hour. The mean of these two computed values is 13, while the true observed value was taken as 12 miles per hour.

Again, as each station can, from its local observations, also find its own $\frac{dp}{dt}$, or rate of fall at any instant, it follows from (2) that by the exchange of telegrams, each station can calculate its own gradient at the given instant, or $\frac{dp}{dr} = \frac{dp/dt}{dr/dt}$, quite free from any theory.

Consequently each station can compute its

$$r = \frac{\Delta p}{2 \frac{dp}{dr}} \quad (5)$$

In words, *the distance of the calm area at any time is the fall below the mean divided by twice the gradient.*

Thus for the storm of 1903, at Montego Bay, we have found that $\frac{dp}{dt}$ was 0.083 at 7 a. m., but as $\frac{dr}{dt}$ was 20, therefore, $\frac{dp}{dr}$ was 0.00415; and by equation (5)

$$r = 40.$$

The true observed value was 45, or thereabouts.

There is no need to proceed any further at present with the mathematical part of this investigation, but I think that much valuable information might be obtained by the discussion of a large number of tropical cyclones on the plan indicated in this article. We want to know more about a and c , and when and why Δp breaks away from the curve and follows the tangent.

As to the practical part of this inquiry, I do not know that observers can do better during the approach of a cyclone, while waiting for time to pass and further exchange of telegrams, than to put their observations into the forms indicated above. Equation (3) has often saved me anxiety, and I have been able to send the second reassuring general telegram, "not coming our way," after the first general warning to the islands had been issued some hours previously.

ON THE CONDITIONS DETERMINING THE FORMATION OF CLOUD-SPHERES AND PHOTOSPHERES.

By ARTHUR W. CLAYDEN, M. A.

[From the Monthly Notices of the Royal Astronomical Society, December, 1905.]

In the course of an investigation of the conditions under which clouds may be formed in our own atmosphere certain considerations presented themselves which seem equally ap-

plicable to the conditions which determine the position of a stellar photosphere in the mass of a star.

As the spectrum of a star is to a great extent dependent upon the position of the photosphere it seems possible that a survey of these points may help to clear up some of the difficulties attending the interpretation of spectral details.

To begin with it is necessary to ask that it may be taken for granted—

1. That the photosphere of a star is the upper surface of a stratum of clouds.

2. That those clouds are caused by the condensation of some substance from the state of vapor to that of small solid or liquid particles.

3. That the condensation is due to cooling produced by expansion brought about by the ascent of vapor-charged convection currents.

4. That the cooling effect of expansion follows the same general thermodynamic law as is the case in our own atmosphere.

It is true that under the high pressures and temperatures of a star the gradation of temperature may be considerably modified. The transference of heat from one stratum to a higher by conduction and radiation should tend to equalize temperatures, but the increased viscosity due to pressure should tend in the opposite direction. Hence, a curve showing the relations of temperature and pressure is not likely to differ very greatly from one plotted in accordance with the expression

$$\frac{\log t - \log t'}{\log p - \log p'} = \frac{\gamma - 1}{\gamma}$$

in which t is the absolute temperature at a pressure p , and t' is the absolute temperature at the reduced pressure p' ; γ is, of course, the ratio of the two specific heats.

In order to argue from the known to the unknown, let us first consider the case of planetary bodies surrounded by an atmosphere consisting wholly of water, a substance whose temperature-pressure relations are well understood.

At temperatures far below freezing point ice gives off vapor which exerts a certain maximum pressure. As the temperature rises this maximum pressure increases more and more rapidly. This goes on until 365° C. is reached, at which point the maximum pressure is 200.5 atmospheres. If at any temperature the pressure be less than the maximum, evaporation will take place; and, conversely, if the pressure exceed the maximum, condensation will follow until the pressure is reduced to that value.

We can then plot a curve (fig. 5, Plate I) showing the maximum pressures for all temperatures. Let this be done, taking temperatures as abscissæ and pressures as ordinates.

We find the curve rises at first very slowly, and at 100° C. it shows a pressure of one atmosphere. It then turns upward more and more rapidly until it reaches 365° C. and 200.5 atmospheres. This is the critical point, and if the temperature be higher no increase of pressure can possibly bring about liquefaction. The curve may be regarded as giving the pressures and temperatures of condensation. If, then, the pressure is greater than 200.5 atmospheres, condensation will be effected at the critical temperature. The curve must therefore turn vertically upward.

Let us now assume that the masses of atmosphere and planet are such that the pressure on the solid surface is 250 atmospheres, and that this surface has a temperature of 400° C. No liquid water can exist on such a surface, but condensation will occur at a certain height.

If the curve of decreasing temperature be plotted, using the expression quoted and giving to γ the proper value for water vapor—namely, 1.3—it will be found that this curve will cut the condensation line very near the critical point, at which temperature and pressure cloud-production will begin.

Suppose, next, that the surface temperature is increased to 500° C. If a similar curve be now plotted, it will be found to cut the condensation line at about 50 atmospheres and a temperature of only 260° C.; that is to say, the effect of increasing the surface temperature of the planet is not only to drive the cloud level farther up in the atmosphere, but to lower the temperature at which its formation begins.

Again, suppose the surface temperature to be 400° C., but the pressure only 50 atmospheres. The curve then cuts the condensation line at about 160° C. and a pressure of less than 10 atmospheres.

It thus appears that the result of suitably diminishing the mass of a planet may be to produce exactly the same effect upon any cloud sphere, by which it is surrounded, as would be brought about by increasing the temperature, or that a hot planet of large mass might present exactly the same features as a cooler and smaller one.

If we imagine the planetary atmosphere to contain other noncondensable gases, this will not affect the conclusions. The changes due to alterations of temperature and pressure will still be in the same direction, and the diagram will serve equally well if we remember that it relates only to the pressures and temperatures of the water vapor present. The actual pressures in the whole atmosphere could be computed by calculating them for the other substances and adding those values.

An inspection of fig. 5, Plate I, shows that no cloud sphere could possibly have a higher base temperature than 365° C. This is one point worth noting.

Next it is evident that if our eyes were so constituted that we could see the radiations emitted from the outer surfaces of such cloud spheres, they would become true photospheres, and if the distances were great enough we should see these supposed planetary bodies as star points. If two of them had their cloud spheres at similar positions in their atmospheres, they should present similar spectral features. We should then class them together, although they might really owe their apparent similarity to their true diversity.

Finally it is obvious that a determination of the temperature of the outer surface of the cloud sphere would be no measure whatever of the temperature of the solid planet beneath.

It is not necessary to point out that these conclusions have an important bearing on the cloud spheres surrounding the actual planets.

It seems only reasonable to attempt to apply them also to the stars.

Now it is not necessary to make any assumption as to the nature of the substance which makes up the cloud particles of a stellar photosphere, nor as it necessary to assume that all photospheres are due to the same body. Whatever the substance concerned its condensation curve would probably present features similar to those of all bodies for which the requisite data are obtainable.

The argument, however, will be clearer if we start with the assumptions that all photospheres are due to the same substance, and that that is the element carbon. It will be easy later on to briefly consider other possibilities.

We have no measurements of the vapor pressures of carbon, but there seems some reason to think, from the phenomena of the electric arc, that the vaporization temperature under one atmosphere pressure is about 3770° absolute.¹ This, then, will correspond to 373° absolute, the boiling point of water under one atmosphere.

If we can get some idea as to the critical temperature we can then draw a curve resembling the known curve for water,

¹ 3770° A. is the same as 3497° C. 373° A. = 100° C. We shall use the abbreviations A. and C. in this sense.—EDITOR.

and feel tolerably sure that our proceedings are reasonable. Messrs. Wilson and Gray estimated the temperature of the solar photosphere at 6900° C. (7173° A.), but this is the upper surface of the cloud stratum, which must be cooler than the base. Hence, if we take 7200° A. as the critical temperature we can feel sure that we are at least within the mark, and may be a long way within it.

The difference between 3770° and 7200° A. is about thirteen times as great as that between 373° A. and the critical temperature for water. Hence, if the condensation curve for carbon is similar to that for water the critical pressure should be about 2600 atmospheres. If we connect the two points thus fixed by a curve resembling that for water it will serve our purpose, since neither the exact temperatures nor pressures are material to the issue, which rests only on the supposition that the vapor pressures and temperatures for the photospheric substance can be represented by some curve of the usual type.

Let us draw the curve, and again, for simplicity, consider only the pressures due to the carbon vapor.

Before drawing the curves to represent the changes of temperature and pressure in the stellar mass we are confronted with another unknown—namely, the value to be assigned to γ . According to the determinations of this quantity yet made, it appears that elementary bodies whose molecules are monatomic give a value 1.66; those whose molecules are diatomic give 1.4 to 1.3; while polyatomic molecules give values decreasing with the complexity, but always greater than unity. At stellar temperatures it seems unlikely that there will be polyatomic molecules, so that if we take 1.5 as a working value we shall not be far wrong. Moreover it will soon be evident that a change in the value of γ will in no way affect our general conclusions, but will only modify the numerical examples which serve the purpose of making the argument clearer.

Let us now imagine six stars in which the pressure of 10,000 atmospheres is reached at $8,000^{\circ}$, $10,000^{\circ}$, $11,850^{\circ}$, $14,950^{\circ}$, $21,500^{\circ}$, and $500,000^{\circ}$ A., respectively. Plot the curves of descending temperature and pressure as we pass outward as before. (See fig. 6, Plate I.) The results may be tabulated thus:

Temperature at $p=10,000$ atmospheres.	Condensation.	
	Pressure.	Temperature.
$^{\circ}$ A.	Atmospheres.	$^{\circ}$ A.
8,000	7,400	7,200
10,000	3,850	7,200
11,850	2,150	7,050
14,950	730	6,250
21,500	130	5,250
500,000	30?	4,400

The table shows that if the initial pressure be far above the critical, the result of increasing the initial temperature is to drive the photosphere into regions of diminished pressure. So long as the pressure of condensation is above the critical, the temperature at which condensation begins remains unaltered; but as soon as this point is passed, and the curved part of the condensation line is reached, the temperature at which the clouds are formed begins to fall; that is to say, the effect of raising the internal temperature is to drive the photosphere farther out, and to cool it.

Again, let us imagine that a temperature of $10,000^{\circ}$ A. is found with pressures of 10,000, 8,000, 6,000, 4,000, 2,000, and 200 atmospheres, respectively, and tabulate the results as before.

The sequence of changes is exactly similar, and the conclusion is that, if the initial temperature is constant, decrease of pressure will produce the same effect as increase of temperature.

Initial pressure, $t=10,000^{\circ}$ A.	Condensation.	
	Pressure.	Temperature.
Atmospheres.	Atmospheres.	$^{\circ}$ A.
10,000	3,850	7,200
8,000	3,050	7,200
6,000	2,150	7,050
4,000	1,150	6,850
2,000	420	5,850
200	30?	4,400

Now the pressure at any point is determined by gravity and the mass of the superincumbent gases. It therefore appears that no carbon photosphere can have a higher temperature than the critical, and that the hotter photospheres must be those most deeply seated or surrounding the most massive stars.

Deep seated photospheres must mean heavy absorption, high photospheres little absorption.

Inspection of the diagram reveals a number of interesting points, and it is easy to see how the spectra of stars should be modified by altering the ratio of temperature and pressure.

Call this $\frac{T}{P}$.

Let us at the outset give this expression a very high value, so high as to make the curve on the scale of our diagram raised very little above the line of no pressure. Such a star would have no photosphere. If the temperature were suitable, small particles of incandescent carbon would be dispersed here and there in a thin mist at a high level. They would be unable to hide the radiation from gases at lower levels or to reverse the light from those among which they were spread. Such stars would be bright-line stars in which the lines would be mixed with a faint continuous spectrum, which would begin with a dim radiance in the yellow-green.

As we reduce the ratio $\frac{T}{P}$ the dim radiance will spread and brighten, since condensation under higher pressure means a higher temperature. The mist stratum of the bright-line star should then pass through denser stages into the condition of a discontinuous stratum of bright photospheric clouds, still high up, though lower than before. The bright clouds will be overlaid by those gases which lie highest, giving a spectrum showing the dark lines due to absorption. But with a discontinuous stratum of cloud there must be convection. Rising currents will make the clouds, descending currents the interspaces. The light, then, from the descending spaces should be bright lines corresponding with those which are reversed above the clouds.

If we now bear in mind the fact that a star spectrum is an integration of the whole light from the stellar surface, it is easy to see that the bright-line spectrum may be brighter than the continuous; it may be equally bright or it may be less brilliant. If the convection movements were slow the respective results would be: bright lines on a continuous background, a continuous spectrum alone, or a simple absorption spectrum. It is, however, not likely that all the lines would behave alike, and the result should generally be a white or helium star showing some lines bright and others reversed.

If we may suppose the convection currents sufficiently rapid, then the bright lines due to descending currents should be shifted toward the red, while other bright lines from deeper layers might be undisturbed. We should then have some bright lines fringed on their more refrangible sides by dark companions.

Decreasing the ratio still further the spaces between the cloudlets will close up until we have the complete helium star.

Further progress will yield a yet brighter and hotter photosphere, sinking step by step beneath stratum after stratum of

gases. As the background gets brighter, helium absorption, having little intensity, becomes less and less obvious. Hydrogen absorption, on the contrary, becomes more and more extensive, until it in turn becomes secondary to the absorption due to metallic vapors.

The curves showing the temperature and pressure at which condensation takes place indicate steadily rising pressure, and rising temperature (and, therefore, intrinsic brightness) until the critical point is reached. From this point no further change in $\frac{T}{P}$ can alter the temperature of the photosphere. As it sinks lower and lower the density of absorption increases. If the metallic-line spectrum of the sun may be supposed to be formed under the conditions represented by the curve *S*, on passing to the next cooler line we might have the denser absorption of *Arcturus*.

If the ratio $\frac{T}{P}$ be still smaller we come to the curve *A*. Here the photosphere is still deeper. The outer parts of the superincumbent gases are much cooler, and compound vapors may be formed. If so, we should expect that the absorption should consist of metallic lines, flutings, and general absorption of the kind known as smoke-veil, for instance, *α Orionis* and *Antares*.

Continuing the process, such a spectrum should grow in intensity until the light of the photosphere should be hidden by superincumbent vapors, or even nonluminous clouds formed by condensed metals and compounds. The last glimpse of the incandescent depths should be a dull red glow.

Such, then, seems to be the normal history of a star.

There is, however, a special case.

Suppose $\frac{T}{P}$ is very high, but that its large value is due to extreme heat, and that *P* is itself large.

The result should be a very dense gaseous nucleus which should give a continuous spectrum, and therefore act as a deeper-seated photosphere whose light would be veiled by absorption, in which that due to carbon vapor would be a conspicuous feature; but metallic lines would also be present, and if the absorption were great, or the intrinsic brightness of the continuous spectrum small, some of the strong metallic lines would stand out as bright lines—carbon stars. Such stars should pass through the normal sequence as a result of declining temperature, which may explain the former redness of *Sirius*.

It thus appears that bright line white stars should be associated with nebulae; that white stars being due to the greatest range of conditions should be most numerous, especially among the smaller stars; that solar stars should be next in order of frequency, and should form a larger proportion of the massive stars; that stars with fluted spectra should be comparatively few in number, and should as a rule be massive. Finally, carbon stars, demanding exceptionally high pressure and temperature, should therefore be rare and vast.

There are several other deductions which may be drawn. First, if a binary is formed by the fission of a single star, and the division is equal, both stars should be white, or both solar, or beyond. If unequal, and differing to a sufficient extent, the smaller would adhere to a Sirian spectrum long after the larger and less cooled had passed into the solar stage or beyond; as, for instance, *β Cygni*.

Second, any determination of the temperature of the photosphere is no guide to the temperature of the star center, neither is the position of the photosphere, as shown by the absorption, much help. It is possible to have a large hot star showing exactly the same spectrum as a much smaller and cooler one. The ratio $\frac{T}{P}$ may be identical.

So far it has been assumed that carbon is the cause of all photospheres. It is, of course, possible that different stars may be differently constituted, and that different elements may play similar parts. But all the evidence of the spectroscope indicates a cosmic distribution of the elements best known to us. Moreover, a moment's thought will show that all that has been said in reference to a carbon photosphere will apply with equal force to any substance whatever. If, then, we can have photospheres formed of some heavier atoms, they should be situated deeper in the mass of the star, and should be overlaid by carbon, which should either form a higher photosphere in turn or should betray its presence by absorption. We ought, then, to have as great a variety of carbon stars as we have of other types. The fact of the rarity of carbon stars is one of the strongest evidences that it is preeminently the photospheric element. However this may be, the main conclusions here set forth remain unaffected, because they hold good for any substance which forms a photosphere, granting only the four postulates with which we started, and that this substance behaves like all others whose condensation curves are known.

THEORY OF THE RAINBOW.¹

By Prof. W. LeCONTE STEVENS, Washington and Lee University. Dated Lexington, Va., May 25, 1906.

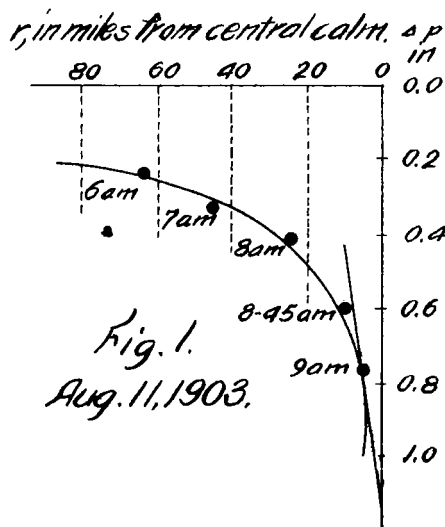
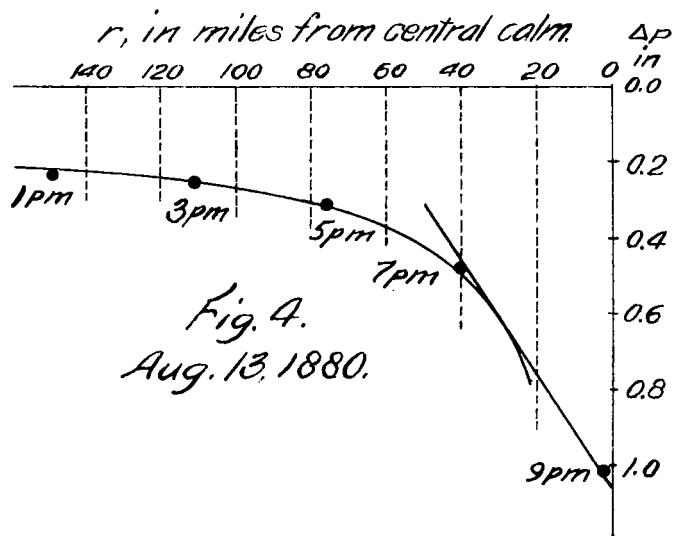
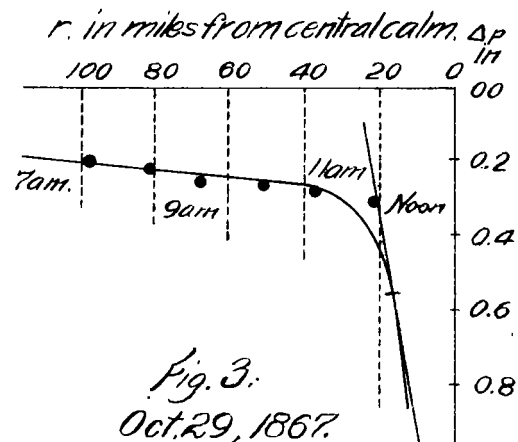
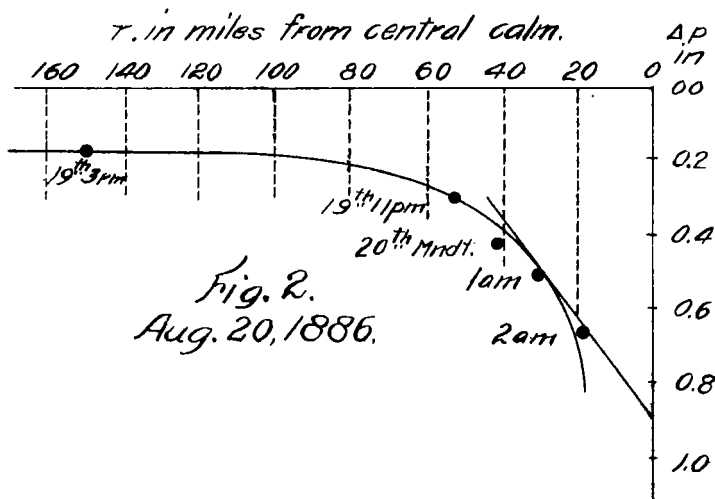
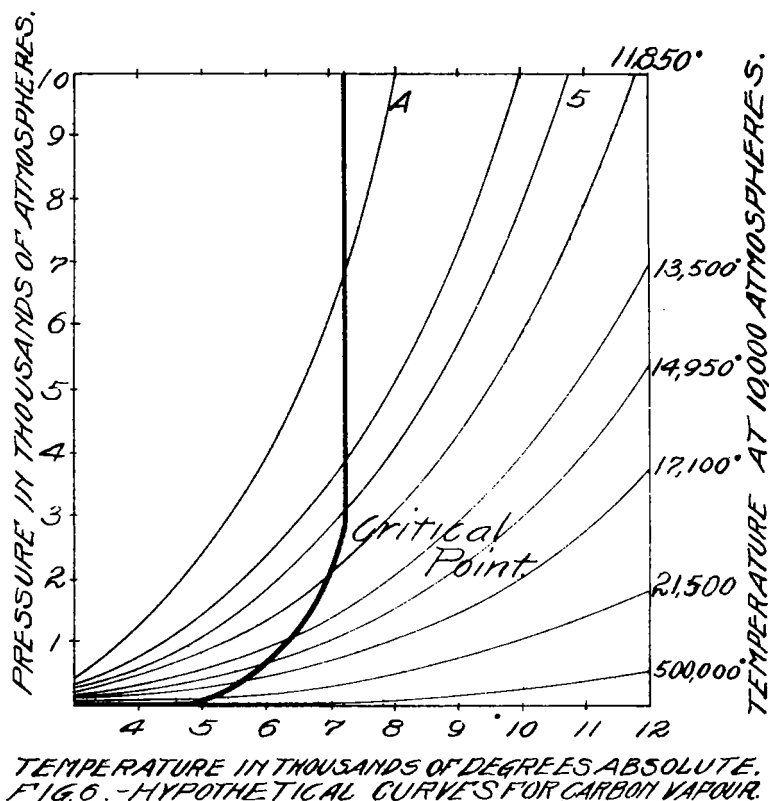
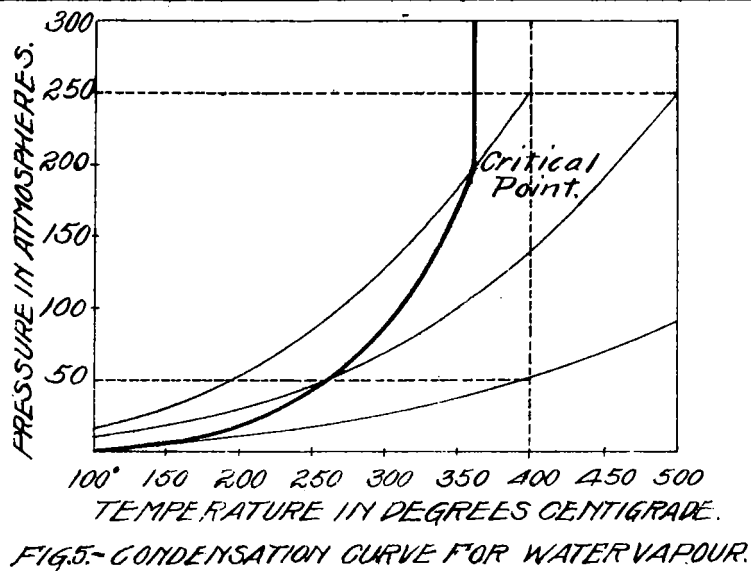
This pamphlet by Doctor Pernter is an attempt to put the complete theory of the rainbow into such form as to involve no application of higher mathematics, especially no application of calculus, and thus to render it suitable for development in the instruction of students who are below the grade of the university or the advanced college class.

To an American teacher, familiar with the limitations found universally in American schools and colleges, the examination of such a paper at once raises the question whether such a demonstration could find a place in any prescribed course in physics in an American college. If not, it would be included in an elective course. This assumption in turn raises the question whether such a subject would probably be attacked voluntarily by any student not already in possession of such elementary knowledge of calculus as to prepare him for the many difficulties that are sure to arise if the study of optics is pursued beyond its elementary stage. Gymnasial instruction in Austria is conducted under conditions somewhat different from those of college instruction in America. The fact that interchange of conditions is not possible, and that Pernter's work would quite surely meet with little appreciation here, does not in any way diminish the merit of what he has done, even if the critical reader is compelled to think that the adaptation to secondary schools is very imperfect on account of the inherent difficulties of the subject.

Pernter begins with the statement that in all schools, high and low, the correct theory of the rainbow is wholly ignored, and that everywhere the "incorrect Descartes's theory of effective rays" (wirksamen Strahlen) is taught, "as if the correct explanation of the rainbow had never been given by Airy". The task which he undertakes is that of presenting the results of Airy's work in a form as simple as the nature of the subject may permit.

At the outset, therefore, it is necessary to dissent from the author's assumption. A geometric theory may be incomplete without being incorrect. From the days of Noah the conditions under which the rainbow appears have been observed and generally known. That light is reflected and refracted at the bounding surface between air and water, was familiar to Ptolemy, but the law of refraction was not discovered until 1621 by Snell. Its correct formulation and publication was subsequently made by Descartes, who died in 1650. Ten

¹ Ein Versuch der richtigen Theorie des Regenbogens Eingang in die Mittelschulen zu verschaffen. Von Dr. J. M. Pernter. Wien, 1898. Selbstverlag.



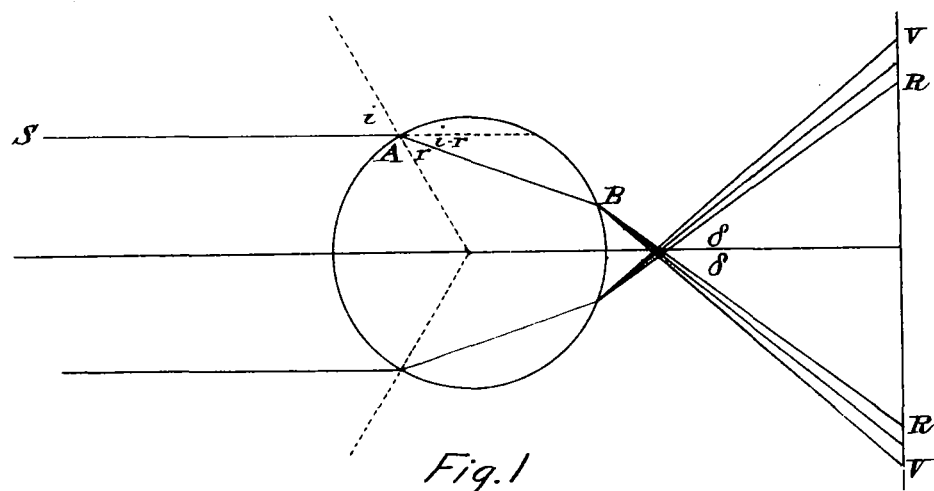


Fig. 1

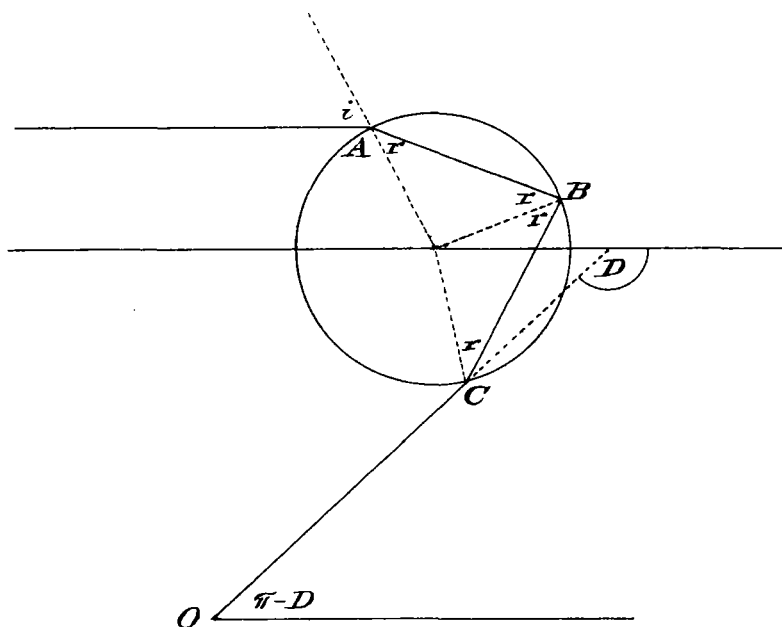


Fig. 2

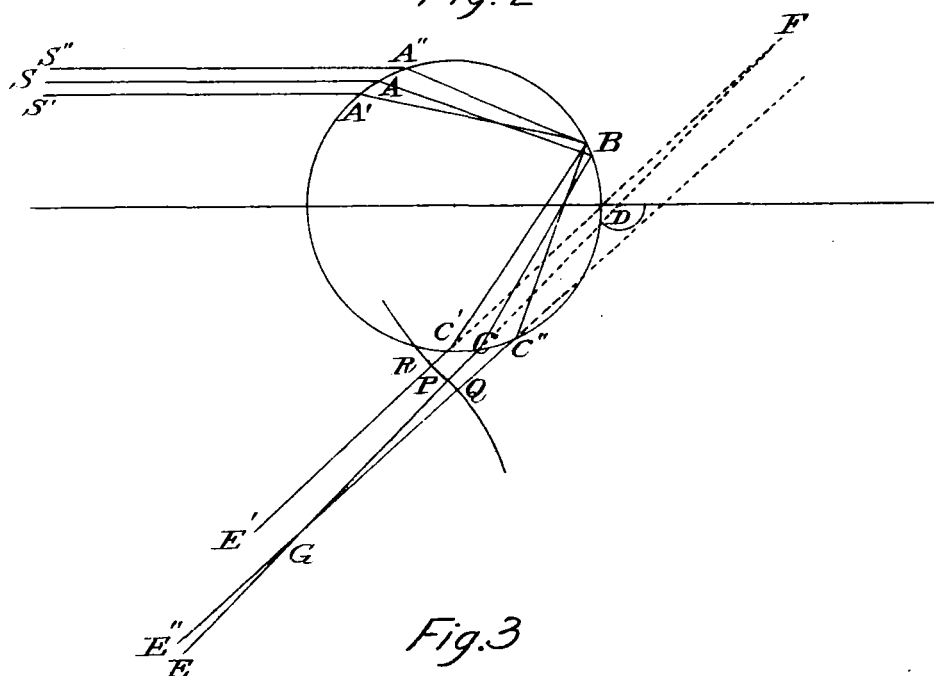


Fig.3

